Amplitude and frequency dependence of the Shapiro steps in the dc- and ac-driven overdamped Frenkel-Kontorova model

Bambi Hu[1,2] and Jasmina Tekić[1,*]
[1]*Department of Physics, Centre for Nonlinear Studies, and The Beijing–Hong Kong–Singapore Joint Centre for Nonlinear and Complex Systems (Hong Kong), Hong Kong Baptist University, Hong Kong, China*
[2]*Department of Physics, University of Houston, Houston, Texas 77204-5005, USA*
(Received 12 April 2006; revised manuscript received 15 February 2007; published 15 May 2007)

The amplitude and frequency dependence of dynamical mode locking phenomena in the dc- and ac-driven overdamped Frenkel-Kontorova model is studied by molecular-dynamics simulations. It was found that the Shapiro steps and the critical depinning force exhibit very complex behavior. The form of amplitude dependence is determined by the frequency of ac force, where the Bessel-type oscillations appear at the high frequencies. With a changing of frequency, after initial increase, the critical depinning force saturates, while the step width remains strongly frequency dependent even at the high frequencies. The dependence of frequency is strongly influenced by the amplitude of ac force where, in the large amplitude regime, the oscillations of the step width and the critical depinning force have been observed at the low frequencies. In the physical processes that stay behind amplitude and frequency oscillations of the step size, an analogy between the influence of amplitude and the period of the ac force is revealed. These oscillations are directly related to the existence and the stability of the interference phenomena in the real systems.

DOI: 10.1103/PhysRevE.75.056608 PACS number(s): 05.45.−a, 45.05.+x

I. INTRODUCTION

Since the first observation of the Shapiro steps [1,2], the phenomena of dynamical mode locking have been the subject of the extensive theoretical and experimental studies in the dissipative dynamics of the many-body systems with competing interactions, such as charge-density or spin-density wave conductors [3–9], vortex lattices [10,11], and Josephson-junction arrays biased by external currents [12–17]. Due to a great complexity of all these macroscopic systems, the attention has always been focused on the simple many-body models that could gain insight into the physics. Among many-body models, the Frenkel-Kontorova (FK) model [18,19] is one of the simplest but still complex enough to capture the essence of many physical and biological phenomena. The numerous theoretical and experimental results in the above systems particularly stimulate the studies of dissipative (overdamped) dynamics of the FK model.

The one-dimensional standard FK model represents a chain of harmonically interacting particles subjected to an external periodic (sinusoidal) substrate potential. It describes different commensurate or incommensurate structures that show very rich dynamical behavior when they are subjected to an external driver. Contrary to the large number of studies of the FK model driven by dc forces, a relatively small number of studies have been devoted to the FK model driven by ac or periodic forces. Dynamics of the dc- and ac-driven FK model is characterized by the appearance of the staircase macroscopic response or the Shapiro steps in the curve for average velocity as a function of the average external driving force $\bar{v}(\bar{F})$. These phenomena are due to interference or dynamical locking of the internal frequency (that comes from the motion of particles over periodic substrate potential) with the frequency of external ac force.

The overdamped dynamics of the both commensurate and incommensurate structures of the one-dimensional standard FK model submitted to dc and ac forces has been studied in detail in the works of Floria *et al.* [20–22]. Using molecular dynamic simulation, for the commensurate structure, they obtained the staircase macroscopic response or the Shapiro steps. The appearance of this quantized increase of the average velocity results from the generation of the coherent, time localized and regularly distributed in-time disturbances (instantons) [20,21,23]. Dynamical mode locking is only possible if the set of ground state is discrete; in the continuum case, the response function $\bar{v}(\bar{F})$ is a continuous strictly increasing function of $\bar{F}$, and there is no mode locking. In the incommensurate structure, the ac-driven dissipative dynamics exhibits the dynamical Aubry transition [22], which represents a borderline between the two different dynamical regimes. Dynamical hull function that describes a driven structure becomes nonanalytical above the transition point, and the result of this is the dynamical locking of the macroscopic response function at certain resonant values. The dynamical mode locking and the dynamical Aubry transition for the commensurate and incommensurate structures, respectively, appear to be one of the universal features of the systems with the competition of time scales in the ac-driven dynamic.

In the present paper, we will examine the influence of the amplitude and frequency of the external ac force on the interference phenomena in the commensurate structures of a one-dimensional standard FK model. We are particularly interested in how the step width and the critical depinning force change with the amplitude and frequency of the ac force. In spite of the extensive studies of the amplitude and frequency dependence of the Shapiro steps in the charge den-

*On leave from Theoretical Physics Department 020, "Vinča" Institute of Nuclear Sciences, P.O. Box 522, 11001 Belgrade, Serbia, Yugoslavia.

sity wave systems and systems of Josephson junction arrays, this has not been so much investigated in the FK model. We obtained that the step width and the critical depinning force exhibit very complex amplitude and frequency dependence, where in the large amplitude regime, the frequency oscillations of the step width and critical depinning force have been observed at the low frequencies. The results have shown that in the physical processes that stay behind the amplitude and frequency oscillations of the step size, the amplitude and period of the ac force have a similar effect.

The paper is organized as follows. The model is introduced in Sec. II. Simulation results are presented and analyzed in Sec. III, where the amplitude dependence is discussed in Sec. III A, and the frequency variations in Sec. III B. Finally, Sec. IV concludes the paper.

II. MODEL

We consider the dissipative (overdamped) dynamics of a series of coupled harmonics oscillators u_l subjected in a sinusoidal substrate (pinning) potential as follows:

$$V(u) = \frac{K}{(2\pi)^2}[1 - \cos(2\pi u)], \tag{1}$$

where K is the pinning strength. The system is driven by dc and ac forces as follows:

$$F(t) = \bar{F} + F_{ac}\cos(2\pi\nu_0 t). \tag{2}$$

The equations of motions are

$$\dot{u}_l = u_{l+1} + u_{l-1} - 2u_l - V'(u_l) + F(t), \tag{3}$$

where $l=-\frac{N}{2},\ldots,\frac{N}{2}$. Equation (3) has been integrated using the fourth-order Runge-Kutta method with the periodic boundary conditions for the commensurate structure with the interparticle average distance (winding number) $\omega=\langle(u_{l+1}-u_l)\rangle$ (ω is rational for the commensurate and irrational for the incommensurate structures). The time step used in the simulations was 0.02τ, and a time interval of 100τ was used as a relaxation time to allow the system to reach the steady state. The force was varied with the step 10^{-4}. The response function $\bar{v}(\bar{F})$, in particular, the step width and the critical depinning force are analyzed for the commensurate structure $\omega=\frac{1}{2}$.

When the system is driven by homogenous periodic force, two frequency scales are present in the system: the frequency ν_0 of the external periodic (ac) force, and the characteristic frequency of the motion over the periodic substrate potential driven by the average force $\bar{F}$. The competition between these two frequency scales can result in the appearance of synchronization phenomena.

If $u_l(t)$ is the solution of Eq. (3), then the transformation

$$\sigma_{i,j,m}\{u_l(t)\} = \{u_{l+i}(t - m/\nu_0) + j\}, \tag{4}$$

produces another solution, where i, j, and m are integers. The solution is called resonant, if there is a triplet of integers such that it is invariant under the symmetry operation as follows:

$$\sigma_{i,j,m}\{u_l(t)\} = \{u_l(t)\}. \tag{5}$$

The average velocity of resonant solution is given by [20]

$$\bar{v} = \frac{i\omega + j}{m}\nu_0. \tag{6}$$

When $m=1$, the resonant solutions and the steps are called harmonic, while when $m>1$, the steps are called subharmonic ($m=2$ for fractional or half-integer steps).

We will consider only the behavior of the harmonic steps. The behavior of the subharmonic steps cannot be studied in the standard FK model since in the FK model with sinusoidal substrate potential, there are no higher order subharmonic steps. For the integer values of ω, only harmonic steps exist [24,25], while the fractional (half-integer) steps appear for the rational noninteger values of ω [20,21,23]. However, these half-integer steps are so narrow compared to the harmonic ones, that they are invisible on the regular plot of the response function $\bar{v}(\bar{F})$. The higher order subharmonic steps can appear in the nonstandard FK model, such as one with an asymmetric deformable substrate potential studied in Ref. [26]. There, large fractional and higher order subharmonic steps appear as a result of the deformation of substrate potential.

III. RESULTS

In the examination of the ac-driven systems, the main interest is always focused on the existence and robustness (structural stability) of the resonant solutions against the changing of the system parameters.

In Fig. 1, the response functions $\bar{v}(\bar{F})$ for the commensurate structure $\omega=\frac{1}{2}$ are presented in two different regimes: for different amplitudes of ac force, Fig. 1(a), and for three different values of frequencies, Fig. 1(b).

Both the amplitude and frequency of the ac force strongly influence the step width and the critical depinning force. The phenomenon of dynamical mode locking is a result of the competition of two forces: the pinning one $V'(u_l)$, and the driving force $F(t)$. As we can clearly see in Fig. 1(a), the steps can only exist in the certain amplitude region, otherwise the system behaves either as a dc-driven dissipative system, when $F_{ac}\to 0$, or as a system of driven free particles, when $F_{ac}\to\infty$. On the curves in Fig. 1(b), we can see the series of the harmonic steps that appear at the integer multiple of $\omega\nu_0$. When frequency decreases, the step width becomes limited by the space between the steps, and with the further decrease of frequency, it must tend to zero. Further, we will present the detailed analysis of the amplitude and frequency dependence of the step width and the critical depinning force.

A. Amplitude dependence of the interference phenomena

In Fig. 2, the width ΔF of the first harmonic step $\bar{v}=0.1$ as a function of ac amplitude F_{ac}, in the three different frequency regimes is presented. The figures are obtained by extracting ΔF from the series of response functions $\bar{v}(\bar{F})$

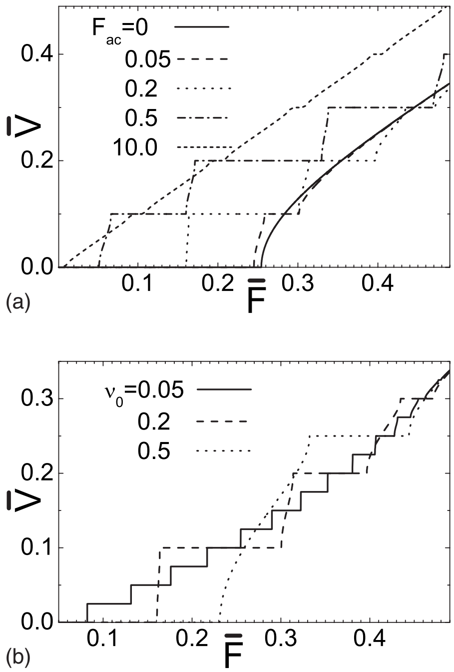

FIG. 1. Average velocity as a function of the average driving force for $\omega=\frac{1}{2}$, $K=4$, in different ac amplitude and frequency regimes: (a) for $\nu_0=0.2$, and $F_{ac}=0$, 0.05, 0.2, 0.5, and 10, and (b) for $F_{ac}=0.2$, and $\nu_0=0.05$, 0.2, 0.5.

made for different amplitudes, and the three different values of frequency.

As we can see, in all frequency regimes, the step width oscillates with ac amplitude, where only at the high frequencies, the oscillations have the Bessel-type form. At the high frequency, in Fig. 2(a), the maximum step width (the first maximum at the curve ΔF versus F_{ac}) is the highest one compared to the cases for lower frequencies, and the envelope of the oscillations decreases with the increase of ac amplitude. With the decrease of ac frequency, in Fig. 2(b), the maximum step width is reduced, while the period of oscillations strongly decreases, and the oscillations start slowly to change from the Bessel-type form. At the very low ac frequency, in Fig. 2(c), the maximum step width and the period of oscillations are very low. The first maximum is not so pronounced, and the oscillations have completely lost the Bessel-type form.

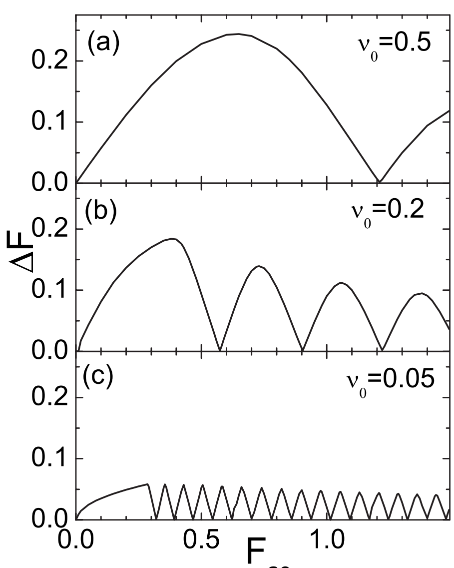

FIG. 2. The width of the first harmonic step $\bar{v}=\omega\nu_0$ as a function of the ac amplitude for $\omega=\frac{1}{2}$, $K=4$, and different values of the ac frequency: (a) $\nu_0=0.5$, (b) $\nu_0=0.2$, and (c) $\nu_0=0.05$.

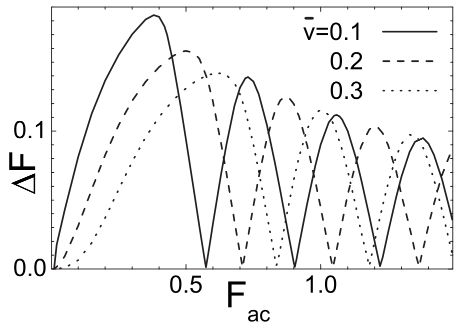

FIG. 3. The widths of the first three harmonic steps $\bar{v}=0.1$, 0.2, and 0.3 as a functions of the ac amplitude for $\omega=\frac{1}{2}$, $K=4$, and $\nu_0=0.2$.

Variations of the step width with ac amplitude for the first three harmonics ($\bar{v}=\frac{i}{m}\omega\nu_0$, where $\frac{i}{m}=\frac{1}{1},\frac{2}{1},\frac{3}{1}$, and $\nu_0=0.2$) are shown in Fig. 3. The maximum step width is the highest for the first harmonic $\bar{v}=0.1$, and it decreases with the harmonic order, while the initial increase becomes more gradual. At the high ac amplitudes, the oscillations for all harmonics become equal, and the phase difference between the curves for even and odd harmonics is 180°.

Application of the ac force on the dc-driven system strongly influences the critical depinning force F_c. In Fig. 4, the critical depinning force as a function of ac amplitude, in

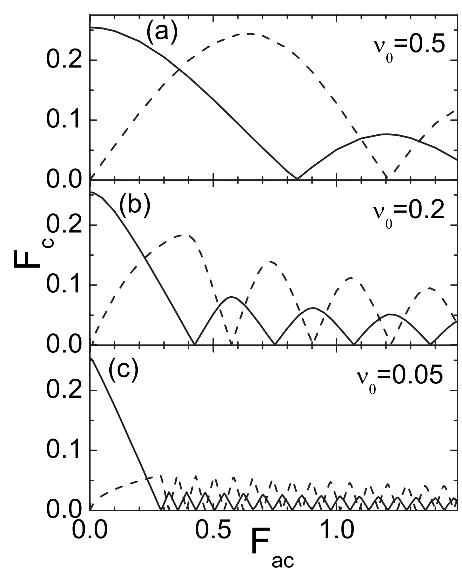

FIG. 4. The critical depinning force as a function of the ac amplitude, for $\omega=\frac{1}{2}$, $K=4$, and different values of the ac frequency: (a) $\nu_0=0.5$, (b) $\nu_0=0.2$, and (c) $\nu_0=0.05$. The dashed curves represent the first harmonic step widths ΔF.

the three different frequency regimes is presented.

As we can see from Fig. 4 and also from Fig. 1(a), when $F_{ac}=0$, F_c reaches the threshold value for the dc-driven system: $F_{c0}=0.2544$. With the increase of ac amplitude, the critical force shows oscillatory behavior in all three frequency regimes. The maxima and the envelope of the oscillations are the highest in Fig. 4(a), for $\nu_0=0.5$. As the frequency decreases for $\nu_0=0.2$, in Fig. 4(b), the maxima and the envelope of the oscillations are reduced, while the period is decreasing. At the low frequency $\nu_0=0.05$, in Fig. 4(c), the maxima and the period of the oscillations are drastically suppressed. In all three cases, we have also shown ΔF of the first harmonic by a dashed curve. The period of oscillations for ΔF and F_c are the same, and according to the results in Figs. 3 and 4, the minima of the F_c curve will correspond to the maxima of the ΔF curve for odd harmonics, and to the minima of the ΔF curve for even harmonics.

The oscillatory behavior of the step size is determined by the ac force. The ac force induces additional polarization energy into the system that is different from zero (less than zero) only when the velocity (internal frequency) reaches the resonant values, in the same time, the average pinning force will also be different from zero. At the resonance, the system will get locked since the average pinning energy of the locked state (on the step) is lower than in the unlocked state, and as $\bar{F}$ increases, it will stay locked until the pinning force can cancel the changes of $\bar{F}$. The size of a step oscillates with ac amplitude due to back and forward displacement of particles induced by the ac force, where the ac amplitude determines how much this motion is retarded [4,6]. For the values of the ac amplitude that correspond to the first maximum, particles will spend most of the time pinned, and then hop to the next well, while for the values at the second maximum, particles will jump one site back and two forward. As the ac amplitude increases, the particles will hop between the wells that are more and more distant while staying less and less time pinned, and consequently, the step width will decrease. Our results show that these Bessel-type oscillations in Figs. 2(a) and 2(b), will be present only at the large frequencies, when the period of the ac force is small. With the decrease of frequency in Fig. 2(c), due to a longer period, even for the same value of ac amplitude, the step size will be smaller since the particles can hop between more distant wells. Displacement between more distant sites will appear only when the ac amplitude is high enough or the period is long enough.

The variations of the step width and the critical depinning force with ac amplitude have been extensively studied in the charge-density wave systems [4,6], vortex lattices, and the systems of Josephson junction arrays [12–17,27,28]. In the charge-density wave systems, the analytical results in the form of Bessel function are obtained in the high frequency limit, in the single coordinate model [6,8]. These analytical results have also been in good agreement with experimental studies [3,4,6] (some early experiments did not give evidence of such oscillations [29,30]; according to them, the step width first increases and then decreases with the ac amplitude). In the systems of Josephson junction arrays, in Refs. [15–17,27], Bessel-type oscillations are obtained by applying a single junction model. The amplitude dependence in our work was analyzed in all frequency regimes, and it was found to be more complex than the commonly obtained Bessel-type oscillations in single coordinate or single junction models, where the influence of frequency has not been studied. In Ref. [31], the influence of frequency on amplitude dependence of steps has been studied in the two-dimensional Josephson junction arrays. Contrary to the single junction model, very complex amplitude dependence has been obtained, with the anomalous oscillations of the step width, and the very sharp roll-off at the high frequencies. In the standard FK model, as our results show, there are no anomalous oscillations. They could probably appear in the FK model studied in Ref. [26], where due to deformations of potential, particles have different energy, and the system does not behave uniformly (due to deformation, the system may split in two or more subsystems that behave differently; while one is still locked, the other may be already moving and this may cause more complex and anomalous dependence).

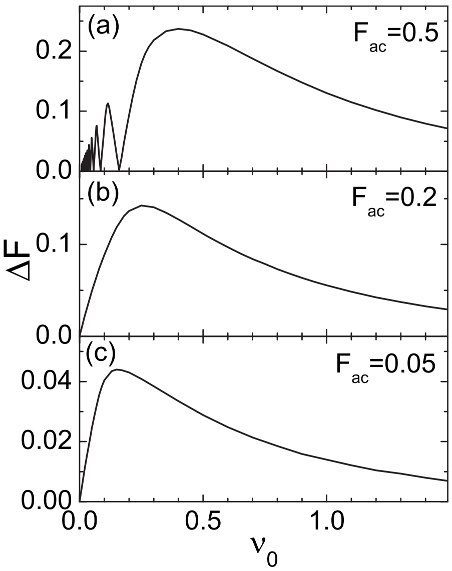

FIG. 5. The width of the first harmonic step $\bar{v}=\omega\nu_0$ as a function of the ac frequency, for $\omega=\frac{1}{2}$, $K=4$, and different values of the ac amplitude: (a) $F_{ac}=0.5$, (b) $F_{ac}=0.2$, and (c) $F_{ac}=0.05$.

B. Frequency dependence of the interference phenomena

The width of the first harmonic step as a function of the ac frequency ν_0 in the three different amplitude regimes is presented in Fig. 5.

In all amplitude regimes, as the frequency increases, the step width first gradually increases, reaching its maximum, and then slowly decreases towards zero at the high frequencies. However, in Fig. 5(a), at $F_{ac}=0.5$, we have observed a very interesting phenomenon. At the very low frequency $\nu_0 < 0.16$ the step width oscillates with the ac frequency, where the maxima and the envelope of the oscillations gradually increase as the frequency increases. These oscillations disappear with a further increase of frequency, where ΔF increases, and then slowly decreases to zero at the very high frequencies. As the ac amplitude decreases, for $F_{ac}=0.2$ in Fig. 5(b), and for $F_{ac}=0.05$ in Fig. 5(c), this low frequency

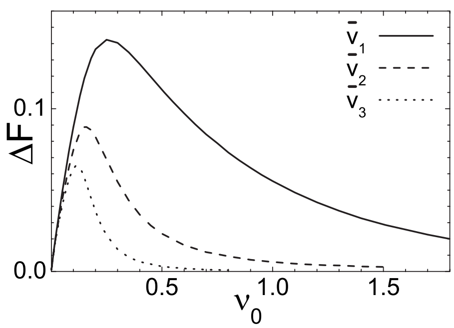

FIG. 6. The width of the first three harmonic steps $\bar{v}_i = \frac{i}{1}\omega\nu_0$, $i=1,2,3$ as a function of the ac frequency for $\omega=\frac{1}{2}$, $K=4$, and $F_{ac}=0.2$.

oscillatory behavior disappears. The initial increase becomes less gradual, while the maximum of the ΔF curve decreases.

The widths of the first three harmonic steps $\left(\bar{v}_i=\frac{i}{1}\omega\nu_0,\ i=1,2,3\right)$ as a function of the ac frequency are shown in Fig. 6.

We can see that the maximum step width is the highest one for the first harmonic, and it considerably decreases with the increase of the harmonic order. At the high frequencies, the higher harmonic steps are completely suppressed.

The critical depinning force F_c as a function of ac frequency, in the three different amplitude regimes is presented in Fig. 7.

In all three amplitude regimes at the low frequencies, F_c first increases as the frequency increases. With the further increase of frequency, it saturates to the frequency independent threshold value for the dc-driven system F_{c0} (represented by the dotted curve). In the high amplitude regime in Fig. 7(a), we again observed the oscillatory behavior at very low frequencies (on the same figure the width of the first

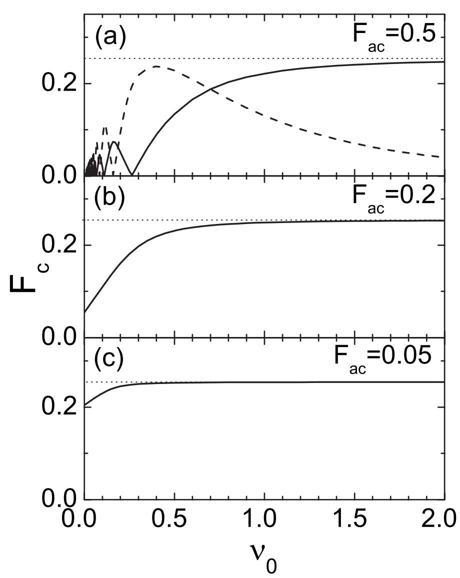

FIG. 7. The critical depinning force as a function of the ac frequency for $\omega=\frac{1}{2}$, $K=4$, and different values of the ac amplitude: (a) $F_{ac}=0.5$, (b) $F_{ac}=0.2$, and (c) $F_{ac}=0.05$. The dashed curve represents the first harmonic step width ΔF. The dotted curve is the threshold value for the dc-driven system: $F_{c0}=0.2544$.

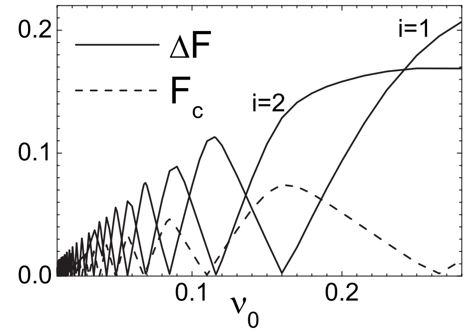

FIG. 8. The widths of the first two harmonic steps ΔF (solid curves) and the critical depinning force F_c (dashed curve) as a function of the ac frequency, in the large amplitude regime, $F_{ac}>F_{c0}$, for $\omega=\frac{1}{2}$, $K=4$, and $F_{ac}=0.5$.

harmonic step from Fig. 5(a) is shown by a dashed curve). With the decrease of the ac amplitude in Fig. 7(b), the oscillatory behavior disappears. When $\nu_0\to 0$, for $F_{ac}=0.2$ in Fig. 7(b), the critical depinning force goes towards the value $F_{c0}-F_{ac}=0.0544$. As the frequency increases, F_c saturates faster to the dc threshold value compared to the case in Fig. 7(a). At the very low amplitude in Fig. 7(c), in the zero frequency limit the critical depinning force reaches the value $F_c=0.2044$, while as the frequency increases, it saturates very quickly to F_{c0}. The oscillatory behavior at the low frequencies has been observed only when $F_{ac}>F_{c0}$. The step widths of the first $(i=1)$ and second harmonics $(i=2)$, and the critical depinning force as a function of the ac frequency are presented in Fig. 8.

As in the case of the step variation with the ac amplitude in Sec. III A, we can clearly see that the maxima of step width for odd harmonics corresponds to the minima of the critical depinning force. We have performed the simulations also for $F_{ac}=F_{c0}$, and obtained that F_c goes to zero when $\nu_0\to 0$. On the other side, if F_{ac} increases, the oscillatory behavior will spread more and more towards the higher frequencies, and for the very large ac amplitudes $(F_{ac}\gg F_{c0})$, the oscillatory behavior will dominate.

These low frequency oscillations are even better revealed in Fig. 9, where the step width is plotted versus period $\left(\frac{1}{\nu_0}\right)$ in the high and low amplitude regimes.

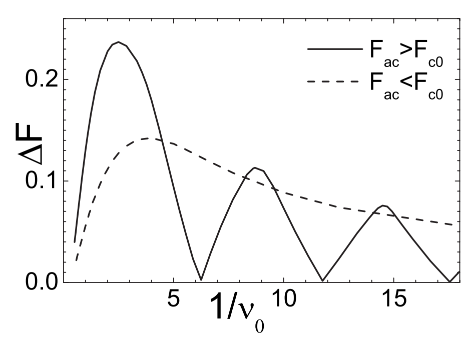

FIG. 9. The widths of the first harmonic step ΔF as a function of $(1/\nu_0)$, for $\omega=\frac{1}{2}$ and $K=4$, in the large $[F_{ac}=0.5,\ F_{ac}>F_{c0}$ (solid curve)] and small $[F_{ac}=0.2,\ F_{ac}<F_{c0}$ (dashed curve)] amplitude regimes.

As we can see, there is a great difference between two amplitude regimes, where for $F_{ac}>F_c$ the form of the curve is very similar to the Bessel oscillations.

These low frequency oscillations that we have observed when $F_{ac}>F_{c0}$ are the result of the simultaneous competition and contributions of the dc and ac component of $F(t)$ to the pinning energy. When $F_{ac}>F_{c0}$, at the low frequencies the ac contribution, which is responsible for the appearance of these oscillations, will dominate in the pinning energy. As in the case of amplitude dependence, Sec. III A, the oscillations appear due to the backward and forward motion of particles induced by ac force. At the low frequencies, due to a very long period, the particles will move between the sites that are further apart, and consequently spend less time pinned, which will result in a smaller step size. As the frequency increases, the displacement will be between closer and closer sites, and at the value of frequency that corresponds to the maximum step width, the particles will spend most of the time pinned, and then they will hop to the next site. The oscillations will exist until the frequency reaches the value for which the dc contribution will cancel the ac contribution (Shapiro steps could be also produced by changing ν_0 and keeping $\bar{F}$ constant).

The interference phenomena have been the subject of many theoretical and experimental studies, however, contrary to the very extensive studies of amplitude dependence, a relatively small number of studies have been devoted to the dependence of frequency. The frequency dependence of the Shapiro steps, and the physics behind interference phenomena are still a matter of debate. In the CDW systems, two competing and fundamentally different theories have been proposed, and interestingly, they have both been equally supported by different experiments [4,32]. According to the classical approach [32,33], which considers a deformable charge elastic medium with the internal degrees of freedom, the step width and the critical depinning force should be strongly frequency dependent, and after an initial increase, decrease to zero at the high frequencies. In contrast, in the other theoretical approach based on the tunneling theory [4–6], where the CDW conductor is treated as a macroscopic quantum system, tunneling of the CDW between the pinned states results in a frequency independent mode locking at the high frequencies. In Refs. [4,8], using a simple single coordinate model motivated by the tunneling theory, it was proved analytically that the maximum step width is proportional to the magnitude of the fundamental component of the effective pinning force that is independent of frequency at the high frequencies. These results are in good agreement with some experiments, and according to Ref. [4], the reduction of degrees of freedom during mode locking might be the cause of frequency independence.

In the systems of Josephson-junction arrays, according to Refs. [15–17,27], the widths of harmonic steps follow the behavior of the single junction model. After a gradual increase at the high frequencies, they saturate to the frequency independent value; meanwhile the widths of subharmonic and fractional steps decrease towards zero due to a departure from single junction behavior. On the other side, in some other theoretical and experimental works [31,34,35], the amplitude and frequency dependence significantly different from the single junction case, and the disappearance of steps at the high frequencies have been observed (single junction models do not work well if the system is disordered [35]).

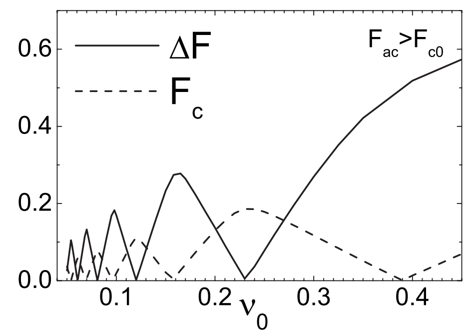

FIG. 10. The widths of the first harmonic step ΔF and the critical depinning force as a function of (ν_0) in the high amplitude regime $(F_{ac}>F_{c0})$ for $\omega=1$, $K=4$, and $F_{ac}=1.2$.

In our work, we have considered an overdamped classical many-body model, and as in other systems with many degrees of freedom the steps will remain strongly frequency dependent and disappear at the high frequencies. Since we have obtained these low frequency oscillations in one system with many degrees of freedom, in order to examine whether these oscillations exist in the single coordinate models, we will then analyze the commensurate structure with the winding number $\omega=1$, for which the FK model reduces to the single particle model [20]. In Fig. 10, the frequency oscillations of the step width and the critical depinning force for the commensurate structure $\omega=1$ in the high amplitude regime are presented.

According to these results we can conclude that the frequency oscillations of the Shapiro steps will appear when $F_{ac}>F_{c0}$ in any commensurate structure, and irrespectively of the number of the degrees of freedom in the system.

Besides the step width and the critical depinning force, the ratio of the second and first harmonic step widths is often examined in the realistic systems. According to the single coordinate model [4], this ratio is nearly constant, indicating again the frequency independence of interference phenomena. We have also examined the frequency dependence of the ratio of the second and first harmonic step widths. In Fig. 11, the results obtained for $\omega=\frac{1}{2}$ in the two amplitude regimes

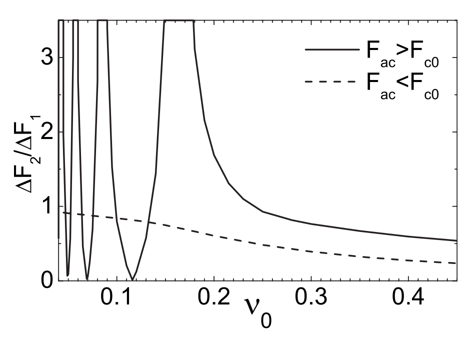

FIG. 11. Ratio of the second and first harmonic step widths as a function of the ac frequency for $K=4$, $\omega=\frac{1}{2}$, and $F_{ac}=1.2$ ($F_{ac}>F_{c0}$, solid curve), and $F_{ac}=0.5$ ($F_{ac}<F_{c0}$, dashed curve).

$F_{ac} < F_{c0}$ and $F_{ac} > F_{c0}$ are presented (similar results are also obtained for $\omega = 1$).

When $F_{ac} < F_{c0}$, the ratio $\frac{\Delta F_2}{\Delta F_1}$ decreases with the increasing of frequency. However, when $F_{ac} > F_{c0}$, due to the appearance of the oscillatory behavior at the low frequencies, very sharp peaks will appear at the points where ΔF_1 goes to zero (the heights of the peaks are more than 100, and the vertical scale on the plot is from 0 to 4, in order to make both curves visible). At the higher frequencies these singularities will disappear, and the ratio will gradually decrease.

IV. CONCLUSION

In this paper we presented a detailed study of the amplitude and frequency dependence of the dynamical mode-locking phenomena in an overdamped dc- and ac-driven FK model. The presented results have shown that the Shapiro steps (resonant solutions) have a very complex amplitude and, in particular, frequency dependence, where the frequency oscillations of the step width and the critical depinning force have been observed in the high amplitude regime. When the amplitude of the ac force is changing, the step width and the critical depinning force exhibit oscillatory behavior, where the type of oscillations is determined by the ac frequency, and commonly observed the Bessel-type form appears only at the high frequencies. The frequency dependence of the step width is strongly determined by the ac amplitude, where at the point $F_{ac} = F_{c0}$, the behavior of the system completely changes. While when $F_{ac} < F_{c0}$, the steps and the critical depinning force will gradually increase with frequency, where the step width will remain strongly frequency dependent and disappear at the high frequency for $F_{ac} > F_{co}$, the oscillatory behavior will appear at the low frequencies. Presenting the step width as a function of $\frac{1}{\nu_0}$, the analogy between the ac amplitude and the period is reveled. Our results have shown that the increase of the period will have a similar effect on the displacement of particles, and consequently on the step size, as the increase of the ac amplitude. Considering two commensurate structures ($\omega = \frac{1}{2}$ and 1), but still working in the same model, we could examine how the degrees of freedom influence the behavior of the system. These low frequency oscillations have been observed in both commensurate structures, irrespectively of the number of degrees of freedom.

We must note that we did not want to favor or criticize any of these different models that are present in the theoretical description of interference phenomena in the charge-density wave systems and the systems of Josephson-junction arrays, and they all can account well in different situations. However, the fact that one and the same physical phenomena are described by the two fundamentally opposite theoretical approaches (the classical one and the one based on the tunneling theory) means that a detailed understanding is still lacking. In spite of the great success of the single coordinate and single junction models, the description of realistic systems cannot always be so simple, and we believe that our results could bring new insight into the physics of interference phenomena in a realistic system, particularly into still existing debate among different models.

Although the above results have been obtained in one very specific model such as Frenkel-Kontorova, they could be of great importance for all real systems with overdamped motion, and driven by periodic forces such as charge- or spin-density-wave systems, vortex lattices, and the systems of Josephson junction arrays. The phenomena of CDW in solids, which account for the anomalous transport properties, and the studies of Josephson junction arrays, which are motivated by technical applications of the Josephson effects, synchronization phenomena, investigations of flux-flow devices are closely connected to the dissipative dynamics of the FK model [3,20]. In the applications of interference effects (synchronization phenomena), the attention is always focused on their stability, and thereafter, on the conditions that produced maximum step size. The results that we have obtained, especially the frequency oscillations of the steps, are directly related to the existence and stability of the resonant solutions, and from that point, particularly interesting, not only for the experiments on interference phenomena, but for all these situations where periodic forces and dynamical mode locking are involved.

ACKNOWLEDGMENTS

This research was supported in part by the Hong Kong Research Grants Council (RGC) and the Hong Kong Baptist University Faculty Research Grant (FRG).

[1] S. Shapiro, A. R. Janus, and S. Holly, Phys. Rev. Lett. **11**, 80 (1963).
[2] C. C. Grimes and Sidney Shapiro, Phys. Rev. **169**, 397 (1968).
[3] G. Grüner, Rev. Mod. Phys. **60**, 1129 (1988).
[4] R. E. Thorne, J. S. Hubacek, W. G. Lyons, J. W. Lyding, and J. R. Tucker, Phys. Rev. B **37**, 10055 (1988).
[5] R. E. Thorne, W. G. Lyons, J. W. Lyding, J. R. Tucker, and J. Bardeen, Phys. Rev. B **35**, 6348 (1987).
[6] R. E. Thorne, W. G. Lyons, J. W. Lyding, J. R. Tucker, and J. Bardeen, Phys. Rev. B **35**, 6360 (1987).
[7] R. E. Thorne, J. R. Tucker, and J. Bardeen, Phys. Rev. Lett. **58**, 828 (1987).
[8] R. E. Thorne, J. R. Tucker, J. Bardeen, S. E. Brown, and G. Grüner, Phys. Rev. B **33**, 7342 (1986).
[9] G. Kriza, G. Quirion, O. Traetteberg, W. Kang, and D. Jérome, Phys. Rev. Lett. **66**, 1922 (1991).
[10] N. Kokubo, R. Besseling, V. M. Vinokur, and P. H. Kes, Phys. Rev. Lett. **88**, 247004 (2002).
[11] A. B. Kolton, D. Domínguez, and N. Grønbech-Jensen, Phys. Rev. Lett. **86**, 4112 (2001).
[12] H. Sellier, C. Baraduc, F. Lefloch, and R. Calemczuk, Phys. Rev. Lett. **92**, 257005 (2004).

[13] S. E. Hebboul and J. C. Garland, Phys. Rev. B **47**, 5190 (1993).
[14] S. Kim, B. J. Kim, and M. Y. Choi, Phys. Rev. B **52**, 13536 (1995).
[15] S. P. Benz, M. S. Rzchowski, M. Tinkham, and C. J. Lobb, Phys. Rev. Lett. **64**, 693 (1990).
[16] M. S. Rzchowski, L. L. Sohn, and M. Tinkham, Phys. Rev. B **43**, 8682 (1991).
[17] S. J. Lee and T. C. Halsey, Phys. Rev. B **47**, 5133 (1993).
[18] O. Braun and Yu. S. Kivshar, *The Frenkel-Kontorova Model* (Springer, Berlin, 2003).
[19] O. Braun and Yu. S. Kivshar, Phys. Rep. **306**, 1 (1998).
[20] L. M. Floría and J. J. Mazo, Adv. Phys. **45**, 505 (1996).
[21] F. Falo, L. M. Floría, P. J. Martínez, and J. J. Mazo, Phys. Rev. B **48**, 7434 (1993).
[22] L. M. Floría and F. Falo, Phys. Rev. Lett. **68**, 2713 (1992).
[23] M. Inui and S. Doniach, Phys. Rev. B **35**, 6244 (1987).
[24] M. J. Renné and D. Polder, Rev. Phys. Appl. **9**, 25 (1974).
[25] J. R. Waldram and P. H. Wu, J. Low Temp. Phys. **47**, 363 (1982).
[26] B. Hu and J. Tekic, Phys. Rev. E **72**, 056602 (2005).
[27] I. F. Marino, Phys. Rev. B **52**, 6775 (1995).
[28] K. H. Lee and D. Stroud, Phys. Rev. B **43**, 5280 (1991).
[29] A. Zettl and G. Grüner, Phys. Rev. B **29**, 755 (1984).
[30] W. G. Lyons, R. E. Thorne, J. H. Miller, and J. R. Tucker, Phys. Rev. B **31**, 6797 (1985).
[31] M. Octavio, J. U. Free, S. P. Benz, R. S. Newrock, D. B. Mast, and C. J. Lobb, Phys. Rev. B **44**, 4601 (1991).
[32] S. N. Coppersmith and P. B. Littlewood, Phys. Rev. Lett. **57**, 1927 (1986).
[33] Daniel S. Fisher, Phys. Rev. B **31**, 1396 (1985).
[34] T. D. Clark and P. E. Lindelof, Phys. Rev. Lett. **37**, 368 (1976).
[35] K. Ravindran, L. B. Gómez, R. R. Li, S. T. Herbert, P. Lukens, Y. Jun, S. Elhamri, R. S. Newrock, and D. B. Mast, Phys. Rev. B **53**, 5141 (1996).